

Research Article

Modeling of An Automatic System with Hydrostatic Transmission

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Abstract

This paper deals with the aspects of modeling, simulation and control of an automatic control system with hydrostatic transmission (HT). The aim of this research is to design the model of a system based on a specific conditions and requirements. Hydrostatic transmitters as a big power transmitters have a broad utilization as an executive device in systems of automatic control. The foundations for designing of those systems are dynamic analyses for HT and control of his elements.

Keywords: Control, servomechanism, hydrostatic power transmitters.

1. Introduction

Hydrostatic power transmitters are the most used power transmitters to control mechanical systems with high degree of accuracy, which are loaded by roll moment (Likaj R et al,2001)(Abduli et al,1975). Electro-hydraulic servo-systems give the high performance in applications of precise positioning task, due to high power density of hydraulic components and simplicity of control signals using electric and electronics components(Situm Z et al, 1997).

For solving the problem of automatic control in this case the most important characteristics are: high coefficient of utilization (0,8) and requirement for a big forces, which are needed for altering control parameters of the servo-pump. The last characteristic requires designing of an additional positional electro-hydraulic servomechanism, which provides the possibility of using low power control signals for plugging the HT on a system of automatic control.

The model has been developed for the conditions and requirements as follows: external load moment $M_v=824$ Nm; moment of inertia for the load $J_t=78,5$ kgm²; maximal load acceleration $\varepsilon_t=12$ rad/s²; maximal load speed $\dot{\theta}_t=1,5$ rad/s;

working frequencies $f=30$ rad/s, and the precision on positioning to be 0.5%. The numerous system simulations are performed and different controllers tested.

2. Energetic solution and energetic calculation of the system

Based on the requirements and working conditions of the system, we can conclude that the appropriate solution for this case offers electro-hydraulic system., Electro-hydraulic system is shown on Figure 1 [2]. Theoretical power on the servo pump axis is:

$$N_t = \frac{(M_v + J_t \cdot \varepsilon_t) \cdot \dot{\theta}_t}{7500} = 2.659(kW) \quad (1)$$

Real hydraulic power on the output axis of servo pump including leakage on tubes is:

$$N_p = \frac{N_t}{\eta_r \cdot \eta_{hm} \cdot \eta_c} = 3.48(kW) \quad (2)$$

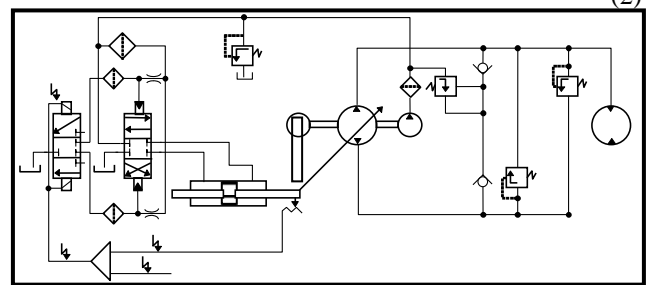


Figure 1. Electro-hydraulic system

3. Dynamic analysis

3.1 Dynamic Analysis of a Hydrostatic Power Transmission System

The values of the gain coefficients used for the selected power transmitter are as follows:

- gain coefficient pump flow,

$$K_{QP} = \frac{\Delta Q_P}{\Delta \gamma} = 1034 (cm^3 / s) / rad \quad (3)$$

- gain coefficient for the power transmitter speed,

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$$K_{\dot{\theta}} = \frac{K_{QP}}{q_m} = 615.5(s^{-1}) \tag{4}$$

- gain coefficient for power transmitter blow,

$$K_1 = 1.22 \cdot 10^{-12} (m^5 / N \cdot s) \tag{5}$$

- fluid volume under pressure in power transmitter,

$$V_t \approx \frac{q_P}{2} + \frac{q_m}{2} + V_c = 410 (cm^3) \tag{6}$$

- roll moment of inertia of the motor,

$$J = J_m + \frac{J_t}{i^2} = 5.79 \cdot 10^{-3} (kg \cdot m^2) \tag{7}$$

3.2. Frequency Analysis for the Automatic Control System

Block diagram of Control System for hydro-static power transmitter is shown on Figure 2. The phase is -128° , this means that system is not stable. To improve stability of the system we should add a gain coefficient on feedback loop of $K_A=0.04$ mA/V. Thus, the overall gain coefficient is as follows:

$$K = K_A \cdot (1/K_P) \cdot K_{\dot{\theta}} = 4.2 \text{ or } K=13dB \tag{8}$$

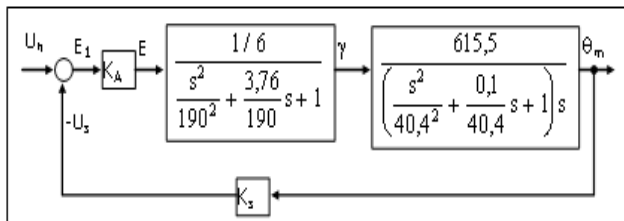


Figure 2. Block diagram of Hydrostatic System

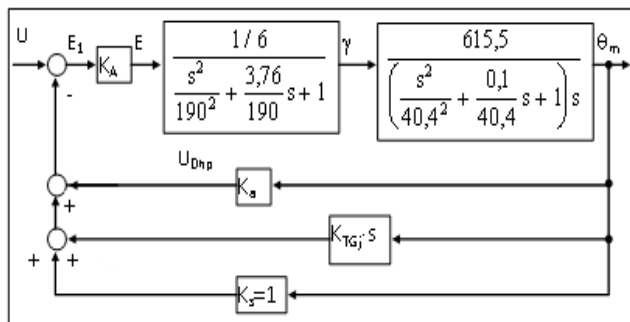


Figure 3. Block diagram of system with feedback compensation.

Peak value of the magnitude fits with the maximum zone of the given working frequency $f=30$ rad/s, this means that system is unstable. The adjustment of the system it can be done by introducing compensation on feedback structure, as it shown on figure 3. Transfer function for the system shown in Figure 3, is as follows:

$$W = \frac{102,5 \cdot K_A \cdot (K_a \cdot s^2 + K_{TGj} \cdot s + 1)}{s \cdot [(s/190)^2 + 3,76s/190 + 1] \cdot [(s/40,4)^2 + 0,1s/40,4 + 1]} \tag{9}$$

Error during translator positioning of the servo system type 1 ($K_{pz} = \infty$), is:

$$e_{pz} = \frac{1}{K_{pz}} = \frac{1}{\infty} = 0 \tag{10}$$

Therefore the given condition which allows error during translator positioning of the system from 0.5% has been performed.

4. Modelling of automatic system

Mathematical model of a hydrostatic system in matrix form is given by:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du \tag{11}$$

where:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{q_m}{J} & 0 & 0 \\ 0 & -\frac{q_m \cdot B}{V_t} & \frac{K_t \cdot B}{V_t} & \frac{K_M \cdot K_{QP} \cdot B}{V_t} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{K_s \cdot K_A \cdot K_{A1} \cdot K_Q}{T_1} & -\frac{K_{TGj} \cdot K_A \cdot K_{A1} \cdot K_Q}{T_1} & -\frac{K_{Dhp} \cdot K_A \cdot K_{A1} \cdot K_Q}{T_1} & -\frac{K_M \cdot K_P \cdot K_{A1} \cdot K_Q}{T_1} & \frac{1}{T_1} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{K_A \cdot K_{A1} \cdot K_Q}{T_1} \end{bmatrix}$$

$$C = [1 \ 0 \ 0 \ 0 \ 0] \text{ and } D = [0]$$

5. Stability analysis

The Lapunov criterion will be used to check whether or not the given system is stable. Starting point for stability analysis is state-space form of system. Matrix A must be definite, x is the state vector containing n state variables. The quadratic form of Lyapunov function is:

$$V(x) = x^T(t) \cdot Q \cdot x(t) \tag{12}$$

where Q must be a positive definite symmetric matrix $n \times n$. General form of Lyapunov matrix equation is:

$$A \cdot Q + Q \cdot A^T = -C \tag{13}$$

Where C is a positive definite symmetric matrix. Usually, matrix Q is taken $Q=I$.

$$C = [0.2137 \ -0.5000 \ -5.5683 \ -0.0073 \ 0.1279; \\ -0.5000 \ 158.5540 \ -0.0176 \ -0.1048 \ -6.9555; \\ -5.5683 \ -0.0176 \ 345.4158 \ 0.2002 \ -108.9819; \\ -0.0073 \ -0.1048 \ 0.2002 \ 0.0119 \ -0.6100; \\ 0.1279 \ -6.9555 \ -108.9819 \ -0.6100 \ 91.4406];$$

From MATLAB, for closed loop is: eig(A-B*K)

$$\text{ans} = 1.0e+004 * \\ -1.1331 \\ -0.0002 + 0.0084i \\ -0.0002 - 0.0084i \\ -0.0001 + 0.0016i \\ -0.0001 - 0.0016i$$

This means that system with compensation on closed loop is stable

Simulations has been done on MATLAB, Simulink block diagram for the system is shown in Figure 4

Simulation parameters selected for this paper are:
 $q_{rm}=1,68\text{cm}^3/\text{rad}$; $B=14000$; $V_t=410\text{cm}^3$;
 $K_f=0,12\text{m}^5/\text{Ns}$; $K_m=0,2\text{rad}/\text{cm}$; $K_{qp}=1034$; $A=1,22\text{cm}^2$;
 $K_s=1$; $K_{DHP}=0,034 \text{ V}/(\text{N}/\text{m}^2)$; $K_{a1}=7,43\text{mA}/\text{V}$;
 $T_1=0,0014\text{s}$; $K_{TGj}=0,007 \text{ V}/(\text{rad}/\text{s})$; $K_p=6\text{V}/\text{rad}$; $U_h=4$ dhe
 $K_A=0,29\text{mA}/\text{V}$.

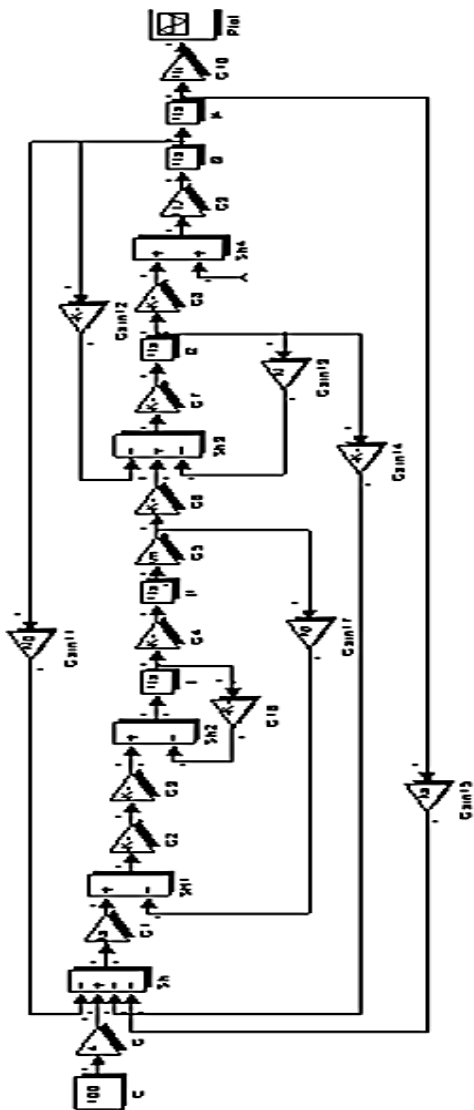


Figure 4. SIMULINK block diagram for simulation process

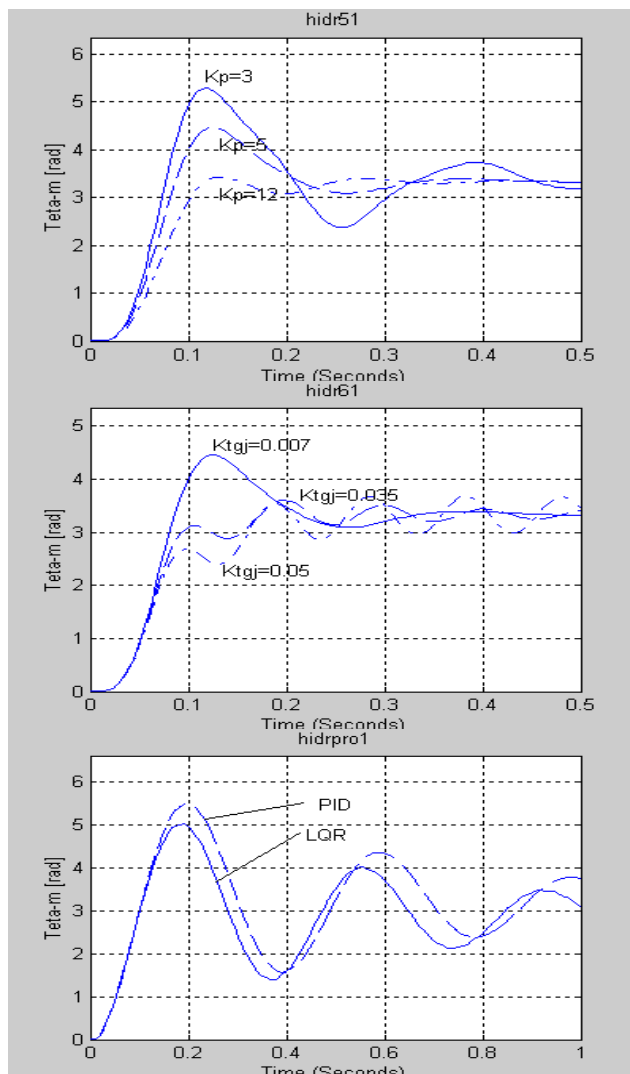
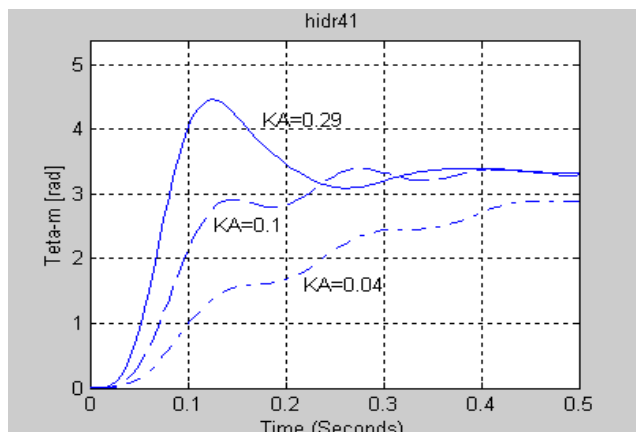


Figure 5. Simulation results for the system obtained by altering the compensation parameters K_A , K_p , K_{tgj} and the comparison of simulation results with LQR and PID Controllers.

Conclusions

The work has demonstrated a possibility of determining the appropriate values of compensation parameters on feedback loop. Using classical methods of linear analysis, we can obtain the conditions of stability and information about how much the system is affected by the change in one parameter. The simulation study shows that the application both controllers LQR and PID (Figure 5), gives a satisfactory results.

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